

Radiative and Semileptonic B Decays Involving Higher K -Resonances in the Final States

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Abstract

We study the radiative and semileptonic B decays involving a spin- J resonant $K_J^{(*)}$ with parity $(-1)^J$ for K_J^* and $(-1)^{J+1}$ for K_J in the final state. Using the large energy effective theory (LEET) techniques, we formulate $B \rightarrow K_J^{(*)}$ transition form factors in the large recoil region in terms of two independent LEET functions $\zeta_{\perp}^{K_J^{(*)}}$ and $\zeta_{\parallel}^{K_J^{(*)}}$, the values of which at zero momentum transfer are estimated in the BSW model. According to the QCD counting rules, $\zeta_{\perp, \parallel}^{K_J^{(*)}}$ exhibit a dipole dependence in q^2 . We predict the decay rates for $B \rightarrow K_J^{(*)}\gamma$, $B \rightarrow K_J^{(*)}\ell^+\ell^-$ and $B \rightarrow K_J^{(*)}\nu\bar{\nu}$. The branching fractions for these decays with higher K -resonances in the final state are suppressed due to the smaller phase spaces and the smaller values of $\zeta_{\perp, \parallel}^{K_J^{(*)}}$. Furthermore, if the spin of $K_J^{(*)}$ becomes larger, the branching fractions will be further suppressed due to the smaller Clebsch-Gordan coefficients defined by the polarization tensors of the $K_J^{(*)}$. We also calculate the forward backward asymmetry of the $B \rightarrow K_J^{(*)}\ell^+\ell^-$ decay, for which the zero is highly insensitive to the K -resonances in the LEET parametrization.

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TABLE I: The data for branching ratios of the radiative and semi-leptonic B decays involving strange mesons.

mode	\mathcal{B} [10^{-6}]	mode	\mathcal{B} [10^{-6}]
$B^+ \rightarrow K^{*+}(892)\gamma$	43.6 ± 1.8 [12–15]	$B^0 \rightarrow K^{*0}(892)\gamma$	43.3 ± 1.5 [12–15]
$B^+ \rightarrow K_2^{*+}(1430)\gamma$	14.5 ± 4.3 [16]	$B^0 \rightarrow K_2^{*0}(1430)\gamma$	12.4 ± 2.4 [16, 17]
$B^+ \rightarrow K_3^{*+}(1780)\gamma$	< 39 [18]	$B^0 \rightarrow K_3^{*0}(1780)\gamma$	< 83 [18]
$B^+ \rightarrow K^{*+}(892)e^+e^-$	$1.42^{+0.43}_{-0.39}$ [2, 5]	$B^0 \rightarrow K^{*0}(892)e^+e^-$	$1.13^{+0.21}_{-0.18}$ [2, 5]
$B^+ \rightarrow K^{*+}(892)\mu^+\mu^-$	$1.12^{+0.32}_{-0.27}$ [2, 5]	$B^0 \rightarrow K^{*0}(892)\mu^+\mu^-$	$1.00^{+0.15}_{-0.13}$ [2, 5, 19]
$B^+ \rightarrow K^{*+}(892)\nu\bar{\nu}$	< 80 [20, 21]	$B^0 \rightarrow K^{*0}(892)\nu\bar{\nu}$	< 120 [20, 21]
$B^+ \rightarrow K_1^+(1270)\gamma$	43 ± 12 [22]	$B^0 \rightarrow K_1^0(1270)\gamma$	< 58 [22]
$B^+ \rightarrow K_1^+(1400)\gamma$	< 15 [22]	$B^0 \rightarrow K_1^0(1400)\gamma$	< 15 [22]
$b \rightarrow s\gamma$	352 ± 25 [23–25]	$b \rightarrow s\ell^+\ell^-$	$4.50^{+1.03}_{-1.01}$ [26–28]

I. INTRODUCTION

The flavor-changing neutral current (FCNC) $b \rightarrow s$ processes suppressed in the standard model (SM) could receive sizable new-physics contributions. Recently BABAR and Belle have shown interesting results on the longitudinal fraction, forward-backward asymmetry and isospin asymmetry of the $B \rightarrow K^*\ell^+\ell^-$ decays [1–6]. Although the data are still consistent with the SM predictions, they favor the flipped-sign c_7^{eff} models [7]. The minimal flavor violation supersymmetry models with large $\tan\beta$ can be fine-tuned to have the flipped sign c_7^{eff} , where the dominant contributions due to the charged Higgs exchange to c_9 and c_{10} are suppressed by $1/\tan^2\beta$ for large $\tan\beta$ [8, 9]. The LHCb is devoted to the B physics studies. Due to the large cross section for $b\bar{b}$ production, the measurement for the rare decays can extend down to 10^{-9} branching ratio. It was estimated by the LHCb collaboration that with a data set of 2 fb^{-2} the $B \rightarrow K^*\ell^+\ell^-$ signal events can be improved by an order of magnitude compared with the present results.

Using the large energy effective theory (LEET) techniques [10], we have formulated the $B \rightarrow K_2^*(1430)$ form factors in the large recoil region [11], and further studied the decays $B \rightarrow K_2^*(1430)\gamma$, $B \rightarrow K_2^*(1430)\ell^+\ell^-$ and $B \rightarrow K_2^*(1430)\nu\bar{\nu}$. In this paper we will generalize to the studies of $B \rightarrow K_J^{(*)}\gamma$, $B \rightarrow K_J^{(*)}\ell^+\ell^-$ and $B \rightarrow K_J^{(*)}\nu\bar{\nu}$ decays

within the SM, where K_J^* and K_J are the spin- J resonances with parities $(-1)^J$ and $(-1)^{J+1}$, respectively. We anticipate to see these modes at LHCb, compared with the current data in Table I [2, 5, 12–30]. In the present study, we will show that the form factors for general $B \rightarrow K_J^{(*)}$ transitions can be parametrized in terms of two independent LEET functions, $\zeta_{\perp}^{K_J^{(*)}}(q^2)$ and $\zeta_{\parallel}^{K_J^{(*)}}(q^2)$ together with the Clebsch-Gordan coefficients, $\alpha_L^{(J)}$ and $\beta_T^{(J)}$. The values of $\zeta_{\perp}^{K_J^{(*)}}(0)$ and $\zeta_{\parallel}^{K_J^{(*)}}(0)$ will be estimated by using the Bauer-Stech-Wirbel (BSW) model [31]. Moreover, we find that branching fractions with higher resonances, $K_J^{(*)}$, becomes smaller not only due to their smaller phase spaces, but also to the smaller $\zeta_{\perp, \parallel}^{K_J^{(*)}}$. Meanwhile, the branching fractions involving $K_J^{(*)}$ with higher spin J will be further suppressed due to smaller Clebsch-Gordan coefficients defined by the polarization tensors of the $K_J^{(*)}$.

There have been a few studies of radiative B decays into higher K -resonances in the literature [11, 32–35]. A discussion for the general cases was given in Ref. [32], where for various processes the authors parameterize the relevant form factors into four Isgur-Wise functions, which are estimated from Isgur-Scora-Grinstein-Wise (ISGW) model [36]. However, they obtained $\mathcal{B}(B \rightarrow K_1(1270)\gamma) < \mathcal{B}(B \rightarrow K_1(1400)\gamma) \simeq (2.4 - 5.2) \times 10^{-5}$, in contradiction to the observation (see Table I). One of the motivations for this work is further to re-examine the other radiative decay channels with higher K -resonances.

This paper is organized as follows. In Sec. II we formulate the $B \rightarrow K_J^{(*)}$ form factors using the LEET techniques. In Sec. III we estimate the LEET form factors, $\zeta_{\perp}^{K_J^{(*)}}(0)$ and $\zeta_{\parallel}^{K_J^{(*)}}(0)$, in the BSW model, and then numerically study the radiative and semileptonic B meson decays into the $K_J^{(*)}$, including the analyses for the forward-backward asymmetries and longitudinal fraction distributions for $B \rightarrow K_J^{(*)}\mu^+\mu^-$. We conclude with a summary in Sec. IV. The derivation of the $B \rightarrow K_J$ form factors is given in Appendix A.

II. $B \rightarrow K_J^*$ FORM FACTORS IN THE LARGE RECOIL REGION

In this section, using the LEET technique, we formulate $B \rightarrow K_J^*$ form factors in the large recoil region. The analogous formulation for $B \rightarrow K_J$ form factors is given in Appendix A. In this paper K_J^* and K_J stand for the higher spin- J K -resonances with parities $(-1)^J$ and $(-1)^{J+1}$, respectively. For simplicity we study in the rest frame of the

B meson (with mass m_B) and assume that the tensor meson K_J^* (with mass $m_{K_J^*}$ and energy E) moves along the z -axis. In the LEET limit, $E, m_B \gg m_{K_J^*}, \Lambda_{\text{QCD}}$, the momenta of the B and K_J^* are given by

$$p_B^\mu = (m_B, 0, 0, 0) = m_B v^\mu, \quad p_{K_J^*}^\mu = (E, 0, 0, p_3) \simeq E n^\mu, \quad (1)$$

respectively. Here $v^\mu = (1, 0, 0, 0)$, $n^\mu = (1, 0, 0, 1)$, and the tensor meson's energy E is given by

$$E = \frac{m_B}{2} \left(1 - \frac{q^2}{m_B^2} + \frac{m_{K_J^*}^2}{m_B^2} \right), \quad (2)$$

with $q = p_B - p_{K_J^*}$.

The polarization tensors $\varepsilon(\lambda)^{\mu_1 \mu_2 \dots \mu_J}$ of the massive spin- J meson with helicity λ that can be constructed in terms of the polarization vectors of a massive vector state with the mass $m_{K_J^*}$

$$\varepsilon(0)^{* \mu} = (p_3, 0, 0, E)/m_{K_J^*}, \quad \varepsilon(\pm 1)^{* \mu} = (0, \mp 1, +i, 0)/\sqrt{2}, \quad (3)$$

are given by

$$\varepsilon(\pm 2)^{\mu \nu} \equiv \varepsilon(\pm 1)^\mu \varepsilon(\pm 1)^\nu, \quad (4)$$

$$\varepsilon(\pm 1)^{\mu \nu} \equiv \sqrt{\frac{1}{2}} [\varepsilon(\pm 1)^\mu \varepsilon(0)^\nu + \varepsilon(0)^\mu \varepsilon(\pm 1)^\nu], \quad (5)$$

$$\varepsilon(0)^{\mu \nu} \equiv \sqrt{\frac{1}{6}} [\varepsilon(+1)^\mu \varepsilon(-1)^\nu + \varepsilon(-1)^\mu \varepsilon(+1)^\nu] + \sqrt{\frac{2}{3}} \varepsilon(0)^\mu \varepsilon(0)^\nu, \quad (6)$$

for $J = 2$ and

$$\varepsilon(\pm 3)^{\mu \nu \rho} = \varepsilon(\pm 1)^\mu \varepsilon(\pm 1)^\nu \varepsilon(\pm 1)^\rho, \quad (7)$$

$$\varepsilon(\pm 2)^{\mu \nu \rho} = \sqrt{\frac{1}{3}} [\varepsilon(0)^\mu \varepsilon(\pm 1)^\nu \varepsilon(\pm 1)^\rho + \varepsilon(\pm 1)^\mu \varepsilon(0)^\nu \varepsilon(\pm 1)^\rho + \varepsilon(\pm 1)^\mu \varepsilon(\pm 1)^\nu \varepsilon(0)^\rho], \quad (8)$$

$$\begin{aligned} \varepsilon(\pm 1)^{\mu \nu \rho} = & \sqrt{\frac{1}{15}} [\varepsilon(\mp 1)^\mu \varepsilon(\pm 1)^\nu \varepsilon(\pm 1)^\rho + \varepsilon(\pm 1)^\mu \varepsilon(\mp 1)^\nu \varepsilon(\pm 1)^\rho + \varepsilon(\pm 1)^\mu \varepsilon(\pm 1)^\nu \varepsilon(\mp 1)^\rho] \\ & + 2\sqrt{\frac{1}{15}} [\varepsilon(\pm 1)^\mu \varepsilon(0)^\nu \varepsilon(0)^\rho + \varepsilon(0)^\mu \varepsilon(0)^\nu \varepsilon(\pm 1)^\rho + \varepsilon(0)^\mu \varepsilon(\pm 1)^\nu \varepsilon(0)^\rho], \end{aligned} \quad (9)$$

$$\begin{aligned} \varepsilon(0)^{\mu \nu \rho} = & \sqrt{\frac{1}{10}} [\varepsilon(0)^\mu \varepsilon(+1)^\nu \varepsilon(-1)^\rho + \varepsilon(+1)^\mu \varepsilon(0)^\nu \varepsilon(-1)^\rho + \varepsilon(+1)^\mu \varepsilon(-1)^\nu \varepsilon(0)^\rho + \\ & \varepsilon(0)^\mu \varepsilon(-1)^\nu \varepsilon(+1)^\rho + \varepsilon(-1)^\mu \varepsilon(0)^\nu \varepsilon(+1)^\rho + \varepsilon(-1)^\mu \varepsilon(+1)^\nu \varepsilon(0)^\rho] + \\ & \sqrt{\frac{2}{5}} \varepsilon(0)^\mu \varepsilon(0)^\nu \varepsilon(0)^\rho, \end{aligned} \quad (10)$$

for $J = 3$, and so on. $\varepsilon(\lambda)^{\mu_1\mu_2\cdots\mu_J}$ is symmetric under interchange of any two of μ_j and μ_k ($1 \leq j, k \leq J$), and satisfies divergence-free conditions $p_{K_J^*,\mu}\varepsilon(\lambda)^{\mu\mu_1\cdots\mu_{J-1}} = 0$, traceless conditions $g_{\mu_1\mu_2}\varepsilon(\lambda)^{\mu_1\mu_2\nu_1\cdots\nu_{J-2}} = 0$, and orthonormal conditions $\varepsilon(h_1)^{\mu_1\mu_2\cdots\mu_J}\varepsilon(h_2)^*_{\mu_1\mu_2\cdots\mu_J} = \delta_{h_1h_2}$.

In the following, we calculate the $\overline{B} \rightarrow \overline{K}_J^*$ transition form factors:

$$\langle \overline{K}_J^* | V^\mu | \overline{B} \rangle, \quad \langle \overline{K}_J^* | A^\mu | \overline{B} \rangle, \quad \langle \overline{K}_J^* | T^{\mu\nu} | \overline{B} \rangle, \quad \langle \overline{K}_J^* | T_A^{\mu\nu} | \overline{B} \rangle, \quad (11)$$

where $V^\mu = \bar{s}\gamma^\mu b$, $A^\mu = \bar{s}\gamma^\mu\gamma_5 b$, $T^{\mu\nu} = \bar{s}\sigma^{\mu\nu}b$ and $T_A^{\mu\nu} = \bar{s}\sigma^{\mu\nu}\gamma_5 b$. In the LEET limit one can easily write down the relevant form factors in terms of the following projectors

$$(\beta_T^{(J)})^{-1} \left(\frac{m_{K_J^*}}{E} \right)^{J-1} [e(\lambda)^{* \mu} - (e(\lambda)^* \cdot v)n^\mu] = \begin{cases} 0 & \text{for } \lambda = \pm 2, \\ \varepsilon(\pm 1)^{* \mu} & \text{for } \lambda = \pm 1, \\ 0 & \text{for } \lambda = 0, \end{cases} \quad (12)$$

$$(\beta_T^{(J)})^{-1} \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \epsilon^{\mu\nu\rho\sigma} e(\lambda)^*_{\nu} n_{\rho} v_{\sigma} = \begin{cases} 0 & \text{for } \lambda = \pm 2, \\ \epsilon^{\mu\nu\rho\sigma} \varepsilon(\pm 1)^*_{\nu} n_{\rho} v_{\sigma} & \text{for } \lambda = \pm 1, \\ 0 & \text{for } \lambda = 0, \end{cases} \quad (13)$$

$$(\alpha_L^{(J)})^{-1} \left(\frac{m_{K_J^*}}{E} \right)^J (e(\lambda)^* \cdot v)n^\mu = \begin{cases} 0 & \text{for } \lambda = \pm 2, \\ 0 & \text{for } \lambda = \pm 1, \\ n^\mu & \text{for } \lambda = 0, \end{cases} \quad (14)$$

$$(\alpha_L^{(J)})^{-1} \left(\frac{m_{K_J^*}}{E} \right)^J (e(\lambda)^* \cdot v)v^\mu = \begin{cases} 0 & \text{for } \lambda = \pm 2, \\ 0 & \text{for } \lambda = \pm 1, \\ v^\mu & \text{for } \lambda = 0, \end{cases} \quad (15)$$

together with $\epsilon^{\mu\nu\alpha\beta}$, v^μ and n^μ , to project the relevant polarization states of the higher K -resonances, where Eqs. (12), (14) and (15) are the vectors, but Eq. (13) the axial-vector. Here $\varepsilon^{0123} = -1$ and we have defined

$$e(\lambda)^{* \mu} \equiv \varepsilon(\lambda)^{* \mu\nu_1\nu_2\cdots\nu_{J-1}} v_{\nu_1} v_{\nu_2} \cdots v_{\nu_{J-1}} = \begin{cases} \alpha_L^{(J)} \varepsilon(0)^\mu \left(\frac{p_3}{m_{K_J^*}} \right)^{J-1} & \text{for } \lambda = 0, \\ \beta_T^{(J)} \varepsilon(\pm 1)^\mu \left(\frac{p_3}{m_{K_J^*}} \right)^{J-1} & \text{for } \lambda = \pm 1, \end{cases} \quad (16)$$

TABLE II: The Clebsch-Gordan coefficients, $\alpha_L^{(J)}$ and $\beta_T^{(J)}$, with $J = 1, 2, \dots, 5$.

J	1	2	3	4	5
$\alpha_L^{(J)}$	1	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{35}}$	$\frac{2}{3}\sqrt{\frac{2}{7}}$
$\beta_T^{(J)}$	1	$\sqrt{\frac{1}{2}}$	$\frac{2}{\sqrt{15}}$	$\frac{1}{\sqrt{7}}$	$2\sqrt{\frac{2}{105}}$

where $\alpha_L^{(J)}$ and $\beta_T^{(J)}$ are the Clebsch-Gordan coefficients of the specific terms of the polarization tensors:

$$\varepsilon(0)^{\mu\nu_1\cdots\nu_n} = \alpha_L^{(J)} \varepsilon(0)^\mu \varepsilon(0)^{\nu_1} \cdots \varepsilon(0)^{\nu_{J-1}} + \text{others}, \quad (17)$$

$$\varepsilon(\pm 1)^{\mu\nu_1\cdots\nu_n} = \beta_T^{(J)} \varepsilon(\pm 1)^\mu \varepsilon(0)^{\nu_1} \cdots \varepsilon(0)^{\nu_{J-1}} + \text{others}, \quad (18)$$

and are given by

$$\alpha_L^{(J)} = \mathcal{J}_{(1,0)(J-1,0)}^{(J,0)} \mathcal{J}_{(1,0)(J-2,0)}^{(J-1,0)} \cdots \mathcal{J}_{(1,0)(1,0)}^{(2,0)}, \quad (19)$$

$$\beta_T^{(J)} = \mathcal{J}_{(1,1)(J-1,0)}^{(J,1)} \mathcal{J}_{(1,0)(J-2,0)}^{(J-1,0)} \mathcal{J}_{(1,0)(J-3,0)}^{(J-2,0)} \cdots \mathcal{J}_{(1,0)(1,0)}^{(2,0)}, \quad (20)$$

with $\mathcal{J}_{(j_1,m_1)(j_2,m_2)}^{(J,M)}$ being the short-hand notations of the following Clebsch-Gordan coefficients

$$\mathcal{J}_{(j_1,m_1)(j_2,m_2)}^{(J,M)} \equiv \langle (j_1 m_1), (j_2 m_2) | JM \rangle. \quad (21)$$

The values of $\alpha_L^{(J)}$ and $\beta_T^{(J)}$ for $J = 1, 2, \dots, 5$ are collected in Table II.

Matching the parities of the matrix elements and using the mentioned Lorentz structures, we can then easily parameterize the form factors to be

$$\langle \bar{K}_J^* | V^\mu | \bar{B} \rangle = -i2E \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \zeta_\perp^{K_J^*(v)} \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho e_\sigma^*, \quad (22)$$

$$\begin{aligned} \langle \bar{K}_J^* | A^\mu | \bar{B} \rangle &= 2E \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \zeta_\perp^{K_J^*(a)} [e^{*\mu} - (e^* \cdot v) n^\mu] \\ &\quad + 2E \left(\frac{m_{K_J^*}}{E} \right)^J (e^* \cdot v) \left[\zeta_\parallel^{K_J^*(a)} n^\mu + \zeta_{\parallel,1}^{K_J^*(a)} v^\mu \right], \end{aligned} \quad (23)$$

$$\begin{aligned} \langle \bar{K}_J^* | T^{\mu\nu} | \bar{B} \rangle &= 2E \left(\frac{m_{K_J^*}}{E} \right)^J \zeta_\parallel^{K_J^*(t)} (e^* \cdot v) \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \\ &\quad + 2E \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \zeta_\perp^{K_J^*(t)} \epsilon^{\mu\nu\rho\sigma} n_\rho [e_\sigma^* - (e^* \cdot v) n_\sigma] \\ &\quad + 2E \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \zeta_{\perp,1}^{K_J^*(t)} \epsilon^{\mu\nu\rho\sigma} v_\rho [e_\sigma^* - (e^* \cdot v) n_\sigma], \end{aligned} \quad (24)$$

$$\begin{aligned}
\langle \overline{K}_J^* | T_A^{\mu\nu} | \overline{B} \rangle &= -i2E \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \zeta_{\perp,1}^{K_J^*(t_5)} \{ [e^{*\mu} - (e^* \cdot v)n^\mu] v^\nu - (\mu \leftrightarrow \nu) \} \\
&\quad -i2E \left(\frac{m_{K_J^*}}{E} \right)^{J-1} \zeta_{\perp}^{K_J^*(t_5)} \{ [e^{*\mu} - (e^* \cdot v)n^\mu] n^\nu - (\mu \leftrightarrow \nu) \} \\
&\quad -i2E \left(\frac{m_{K_J^*}}{E} \right)^J \zeta_{\parallel}^{K_J^*(t_5)} (e^* \cdot v) (n^\mu v^\nu - n^\nu v^\mu).
\end{aligned} \tag{25}$$

Note that the parity of the K_J^* is $(-1)^J$. $\langle \overline{K}_J^* | T^{\mu\nu} | \overline{B} \rangle$ is related to $\langle \overline{K}_J^* | T_A^{\mu\nu} | \overline{B} \rangle$ by the relation: $\sigma^{\mu\nu} \gamma_5 \epsilon_{\mu\nu\rho\sigma} = 2i\sigma^{\rho\sigma}$. Note also that only the K_J^* with polarization helicities ± 1 and 0 contribute to the $\overline{B} \rightarrow \overline{K}_J^*$ transition in the LEET limit, where ζ_{\perp} 's are relevant to K_J^* with helicity $= \pm 1$, and ζ_{\parallel} 's to K_J^* with helicity $= 0$.

We can further reduce the number for the $\overline{B} \rightarrow \overline{K}_J^*$ form factors which are independent, using the effective current operator $\bar{s}_n \Gamma b_v$ (with $\Gamma = 1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu}, \sigma^{\mu\nu} \gamma_5$) in the LEET limit, instead of the the original one $\bar{s} \Gamma b$ [10]. Here b_v and s_n satisfy $\not{v} b_v = b_v$, $\not{s}_n = 0$ and $(\not{v} \not{s}_n / 2) s_n = s_n$. Employing the Dirac identities

$$\frac{\not{v} \not{s}_n}{2} \gamma^\mu = \frac{\not{v} \not{s}_n}{2} (n^\mu \not{v} - i \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \gamma_\sigma \gamma_5), \tag{26}$$

$$\frac{\not{v} \not{s}_n}{2} \sigma^{\mu\nu} = \frac{\not{v} \not{s}_n}{2} [i(n^\mu v^\nu - n^\nu v^\mu) - i(n^\mu \gamma^\nu - n^\nu \gamma^\mu) \not{v} - \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \gamma_\sigma \gamma_5], \tag{27}$$

one can easily obtain the following relations:

$$\bar{s}_n b_v = v_\mu \bar{s}_n \gamma^\mu b_v, \tag{28}$$

$$\bar{s}_n \gamma^\mu b_v = n^\mu \bar{s}_n b_v - i \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{s}_n \gamma_\sigma \gamma_5 b_v, \tag{29}$$

$$\bar{s}_n \gamma^\mu \gamma_5 b_v = -n^\mu \bar{s}_n \gamma_5 b_v - i \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{s}_n \gamma_\sigma b_v, \tag{30}$$

$$\bar{s}_n \sigma^{\mu\nu} b_v = i [n^\mu v^\nu \bar{s}_n b_v - n^\mu \bar{s}_n \gamma^\nu b_v - (\mu \leftrightarrow \nu)] - \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \bar{s}_n \gamma_5 b_v, \tag{31}$$

$$\bar{s}_n \sigma^{\mu\nu} \gamma_5 b_v = i [n^\mu v^\nu \bar{s}_n \gamma_5 b_v + n^\mu \bar{s}_n \gamma^\nu \gamma_5 b_v - (\mu \leftrightarrow \nu)] - \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \bar{s}_n b_v. \tag{32}$$

We can then obtain

$$\zeta_{\perp}^{K_J^*(v)} = \zeta_{\perp}^{K_J^*(a)} = \zeta_{\perp}^{K_J^*(t)} = \zeta_{\perp}^{K_J^*(t_5)} \equiv \zeta_{\perp}^{K_J^*}(q^2), \tag{33}$$

$$\zeta_{\parallel}^{K_J^*(a)} = \zeta_{\parallel}^{K_J^*(t)} = \zeta_{\parallel}^{K_J^*(t_5)} \equiv \zeta_{\parallel}^{K_J^*}(q^2), \tag{34}$$

$$\zeta_{\parallel,1}^{K_J^*(a)} = \zeta_{\perp,1}^{K_J^*(t_5)} = \zeta_{\perp,1}^{K_J^*(t)} = 0. \tag{35}$$

Thus we find that there are only two independent form factors, $\zeta_{\perp}^{K_J^*}(q^2)$ and $\zeta_{\parallel}^{K_J^*}(q^2)$, for the $\overline{B} \rightarrow \overline{K}_J^*$ transition in the large recoil region. In the full theory, the $\overline{B} \rightarrow \overline{K}_J^*$ form

factors are defined as

$$\langle \bar{K}_J^*(p_{K_J^*}, \lambda) | \bar{s} \gamma^\mu b | \bar{B}(p_B) \rangle = -i \frac{2}{m_B + m_{K_J^*}} \tilde{V}^{K_J^*}(q^2) \epsilon^{\mu\nu\rho\sigma} p_{B\nu} p_{K_J^*\rho} e(\lambda)_\sigma^*, \quad (36)$$

$$\begin{aligned} \langle \bar{K}_J^*(p_{K_J^*}, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | \bar{B}(p_B) \rangle &= 2m_{K_J^*} \tilde{A}_0^{K_J^*}(q^2) \frac{e(\lambda)^* \cdot p_B}{q^2} q^\mu \\ &\quad + (m_B + m_{K_J^*}) \tilde{A}_1^{K_J^*}(q^2) \left[e(\lambda)^{* \mu} - \frac{e(\lambda)^* \cdot p_B}{q^2} q^\mu \right] \\ &\quad - \tilde{A}_2^{K_J^*}(q^2) \frac{e(\lambda)^* \cdot p_B}{m_B + m_{K_J^*}} \left[p_B^\mu + p_{K_J^*}^\mu - \frac{m_B^2 - m_{K_J^*}^2}{q^2} q^\mu \right], \end{aligned} \quad (37)$$

$$\langle \bar{K}_J^*(p_{K_J^*}, \lambda) | \bar{s} \sigma^{\mu\nu} q_\nu b | \bar{B}(p_B) \rangle = -2\tilde{T}_1^{K_J^*}(q^2) \epsilon^{\mu\nu\rho\sigma} p_{B\nu} p_{K_J^*\rho} e(\lambda)_\sigma^*, \quad (38)$$

$$\begin{aligned} \langle \bar{K}_J^*(p_{K_J^*}, \lambda) | \bar{s} \sigma^{\mu\nu} \gamma_5 q_\nu b | \bar{B}(p_B) \rangle &= -i\tilde{T}_2^{K_J^*}(q^2) \left[(m_B^2 - m_{K_J^*}^2) e(\lambda)^{* \mu} \right. \\ &\quad \left. - (e(\lambda)^* \cdot p_B) (p_B^\mu + p_{K_J^*}^\mu) \right] \\ &\quad - i\tilde{T}_3^{K_J^*}(q^2) (e(\lambda)^* \cdot p_B) \left[q^\mu - \frac{q^2}{m_B^2 - m_{K_J^*}^2} (p_B^\mu + p_{K_J^*}^\mu) \right], \end{aligned} \quad (39)$$

where

$$e(\lambda)^{* \mu} \equiv \varepsilon(p_{K_J^*}, \lambda)^{* \mu \nu_1 \nu_2 \cdots \nu_{J-1}} p_{B, \nu_1} p_{B, \nu_2} \cdots p_{B, \nu_{J-1}} / m_B^{J-1}, \quad \lambda = 0, \pm 1. \quad (40)$$

Comparing Eqs. (36)-(39) with Eqs. (22)-(25), we obtain

$$\tilde{A}_0^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv A_0^{K_J^*}(q^2) \simeq \left(1 - \frac{m_{K_J^*}^2}{m_B E} \right) \zeta_\parallel^{K_J^*}(q^2) + \frac{m_{K_J^*}}{m_B} \zeta_\perp^{K_J^*}(q^2), \quad (41)$$

$$\tilde{A}_1^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv A_1^{K_J^*}(q^2) \simeq \frac{2E}{m_B + m_{K_J^*}} \zeta_\perp^{K_J^*}(q^2), \quad (42)$$

$$\tilde{A}_2^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv A_2^{K_J^*}(q^2) \simeq \left(1 + \frac{m_{K_J^*}}{m_B} \right) \left[\zeta_\perp^{K_J^*}(q^2) - \frac{m_{K_J^*}}{E} \zeta_\parallel^{K_J^*}(q^2) \right], \quad (43)$$

$$\tilde{V}^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv V^{K_J^*}(q^2) \simeq \left(1 + \frac{m_{K_J^*}}{m_B} \right) \zeta_\perp^{K_J^*}(q^2), \quad (44)$$

$$\tilde{T}_1^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv T_1^{K_J^*}(q^2) \simeq \zeta_\perp^{K_J^*}(q^2), \quad (45)$$

$$\tilde{T}_2^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv T_2^{K_J^*}(q^2) \simeq \left(1 - \frac{q^2}{m_B^2 - m_{K_J^*}^2} \right) \zeta_\perp^{K_J^*}(q^2), \quad (46)$$

$$\tilde{T}_3^{K_J^*}(q^2) \left(\frac{|\vec{p}_{K_J^*}|}{m_{K_J^*}} \right)^{J-1} \equiv T_3^{K_J^*}(q^2) \simeq \zeta_\perp^{K_J^*}(q^2) - \left(1 - \frac{m_{K_J^*}^2}{m_B^2} \right) \frac{m_{K_J^*}}{E} \zeta_\parallel^{K_J^*}(q^2), \quad (47)$$

where we have used $e^{*\mu} \approx (p_{K_J^*}/m_{K_J^*})^{J-1} \tilde{\varepsilon}_{(J)}^{*\mu}$ with $\tilde{\varepsilon}_{(J)}(0)^\mu = \alpha_L^{(J)} \varepsilon(0)^\mu$, $\tilde{\varepsilon}_{(J)}(\pm 1) = \beta_T^{(J)} \varepsilon(\pm 1)^\mu$ and $|\vec{p}_{K_J^*}|/E \simeq 1$.

With the replacement $\varepsilon^\mu \rightarrow \tilde{\varepsilon}_{(J)}^\mu$, we can easily generalize the studies for $B \rightarrow K^* \gamma$, $B \rightarrow K^* \ell^+ \ell^-$ and $B \rightarrow K^* \nu \bar{\nu}$ to the corresponding decays involving resonant strange tensor mesons.

III. NUMERICAL ANALYSIS

The properties of $K_J^{(*)}$ mesons are summarized in Table III. In the following numerical study, we use the values of the parameters listed in Table IV.

A. The determination of form factors and $B \rightarrow K_J^{(*)} \gamma$ Decays

The $B \rightarrow K_J^{(*)} \gamma$ decay widths are given by

$$\begin{aligned} \Gamma(B \rightarrow K_J^{(*)} \gamma) &= \frac{G_F^2 \alpha_{EM}^2 |V_{ts}^* V_{tb}|^2}{32\pi^4} m_{b,\text{pole}}^2 m_B^3 \left(1 - \frac{m_{K_J^{(*)}}^2}{m_B^2} \right)^3 \\ &\times \left| c_7^{(0)\text{eff}} + A^{(1)}(\mu) \right|^2 \left| T_1^{K_J^{(*)}}(0) \right|^2 \left(\beta_T^{(J)} \right)^2. \end{aligned} \quad (48)$$

As for the case with $J = 2$, taking into account the data of $\mathcal{B}(\overline{B}^0 \rightarrow \overline{K}_2^0 \gamma)$ and using $c_7^{(0)\text{eff}} = -0.315$, $A^{(1)} = A_{c_7}^{(1)} + A_{\text{ver}}^{(1)} = -0.038 - 0.016i$ [39], we have obtained [11]

$$T_1^{K_2^{*}(1430)}(0) \simeq \zeta_\perp^{K_2^{*}(1430)}(0) = 0.28 \pm 0.03_{-0.01}^{+0.00}, \quad (49)$$

where the first and second errors are due to uncertainties of the data and the pole mass of the b quark, respectively. In the present paper we use the BSW model [31] to estimate

TABLE III: Properties of resonant $K_J^{(*)}$ mesons (with $J = 1, \dots, 5$) [29], and $B \rightarrow K_J^{(*)}$ LEET form factors calculated in the BSW model [31]. $K_1(1270)$ and $K_1(1400)$ are not considered in this paper (see Refs. [35, 37]). States denoted by “(†)” or “?” are not yet well confirmed. In the present paper we do not take into account 1^3G_3 and 1^3H_4 states.

$K_J^{(*)}$	J^{PC}	$n^{2S+1}L_J$	$m_{K_J^{(*)}}$ [MeV]	$\zeta_{\perp}(0)$	$\zeta_{\parallel}(0)$
$K^*(1410)$	1^{--}	$2^3S_1?$	$1,414 \pm 15$	0.28 ± 0.04	0.22 ± 0.03
$K^*(1680)$	1^{--}	1^3D_1	$1,717 \pm 32$	0.24 ± 0.05	0.18 ± 0.03
$K_2^*(1430)$	2^{++}	1^3P_2	$1,425.6 \pm 1.5$ ($K_2^{*\pm}$)	0.28 ± 0.04	0.22 ± 0.03
			$1,432.4 \pm 1.3$ (K_2^{*0})	0.28 ± 0.04	0.22 ± 0.03
$K_2^*(1980)$ (†)	$2^{+?}$	1^3F_2 or $2^3P_2?$	$1,973 \pm 26$	0.20 ± 0.05	0.14 ± 0.03
$K_3^*(1780)$	3^{--}	1^3D_3	$1,776 \pm 7$	0.23 ± 0.05	0.16 ± 0.03
$K_4^*(2045)$	4^{++}	1^3F_4	$2,045 \pm 9$	0.19 ± 0.05	0.13 ± 0.03
$K_5^*(2380)$ (†)	$5^{-?}$	$1^3G_5?$	$2,382 \pm 24$	0.15 ± 0.05	0.10 ± 0.03
$K_1(1650)$ (†)	$1^{+?}$	2^1P_1 or $2^3P_1?$	$1,650 \pm 50$	0.24 ± 0.05	0.18 ± 0.03
$K_2(1770)$	2^{-+}	1^1D_2	$1,773 \pm 8$	0.23 ± 0.05	0.17 ± 0.03
$K_2(1820)$	2^{--}	$1^3D_2?$	$1,816 \pm 13$	0.22 ± 0.05	0.16 ± 0.03
$K_2(2250)$ (†)	$2^{-?}$	2^1D_2	$2,247 \pm 17$	0.16 ± 0.05	0.11 ± 0.03
$K_3(2320)$ (†)	$3^{+?}$	1^1F_3 or $1^3F_3?$	$2,324 \pm 24$	0.15 ± 0.05	0.10 ± 0.03
$K_4(2500)$ (†)	$4^{-?}$	1^1G_4 or $1^3G_4?$	$2,490 \pm 20$	0.13 ± 0.04	0.09 ± 0.03
$K_5(2600?)$ (†)	$5^{+?}$	1^1H_5 or $1^3H_5?$	$\sim 2,600?$	0.12 ± 0.04	0.08 ± 0.02

the LEET form factors at zero momentum transfer, which are be written by

$$\begin{aligned}
\zeta_{\perp}^{K_J^{(*)}}(0) &= \frac{m_b - m_s}{2E} J, \\
\zeta_{\parallel}^{K_J^{(*)}}(0) &= \left(A_0^{K_J^{(*)}}(0) - \frac{m_{K_J^{(*)}}}{m_B} \zeta_{\perp}^{K_J^{(*)}}(0) \right) \left(1 - \frac{m_{K_J^{(*)}}^2}{m_b E} \right)^{-1}, \quad (50)
\end{aligned}$$

TABLE IV: Input parameters

B lifetime (picosecond)	$\tau_{B^+} = 1.638, \quad \tau_{B^0} = 1.530$
b quark mass	$m_{b,\text{pole}} = 4.79_{-0.08}^{+0.19} \text{ GeV}$
CKM parameter [38]	$ V_{ts}^* V_{tb} = 0.040 \pm 0.001$

where, after integrating out the degrees of freedom of the spins,

$$\begin{aligned}
J &= \sqrt{2} \int d^2 p_T \int_0^1 \frac{dx}{x} \Phi_{K_J^{(*)}}(p_T, x) \Phi_{m_B}(p_T, x), \\
A_0^{K_J^{(*)}}(0) &= \int d^2 p_T \int_0^1 dx \Phi_{K_J^{(*)}}(p_T, x) \Phi_{m_B}(p_T, x).
\end{aligned} \tag{51}$$

Here, for a meson with mass m its wave function can be parameterized as

$$\Phi_m(p_T, x) = N_m \sqrt{x(1-x)} e^{-p_T^2/2\omega^2} e^{-\frac{m^2}{2\omega^2} \left(x - \frac{1}{2} - \frac{m_{q1}^2 - m_{q2}^2}{2m^2} \right)^2}, \tag{52}$$

with N_m being a normalization factor such that

$$\int d^2 p_T \int_0^1 dx \Phi_m^2 = 1, \tag{53}$$

and m_{q1} and m_{q2} the constituent quark masses of the non-spectator and spectator quarks participating in the quark decaying process. We use $\omega = 0.46 \pm 0.05 \text{ GeV}$ and the following constituent quark masses in the model calculation: $m_u = m_d = 0.33 \text{ GeV}$, $m_s = 0.50 \text{ GeV}$, $m_b = 4.9 \text{ GeV}$. The value of ω , which determines the average transverse quark momentum and is approximately the same for mesons with the same light spectator quark [31], is fixed by the $\mathcal{B}(\overline{B}^0 \rightarrow \overline{K}_2^0 \gamma)$ data. The numerical results for $\zeta_{\perp}^{K_J^{(*)}}(0)$ and $\zeta_{\parallel}^{K_J^{(*)}}(0)$ are collected in Table III.

The detailed results for the branching fractions for $B \rightarrow K_J^{(*)} \gamma$ decays are given in Table V. Note that the decay with a heavier meson in the final state has a smaller branching fraction not only due to the smaller phase space and $\zeta_{\perp}^{K_J^{(*)}}(0)$ but also to the Clebsch-Gordan coefficient $\beta_T^{(J)}$ which is smaller for a larger spin J (see Table II). We find

$$\begin{aligned}
&\mathcal{B}(B \rightarrow K^*(1410)\gamma) > \mathcal{B}(B \rightarrow K^*(1680)\gamma) > \mathcal{B}(B \rightarrow K_2^*(1430)\gamma) \\
&> \mathcal{B}(B \rightarrow K_2^*(1980)\gamma) > \mathcal{B}(B \rightarrow K_3^*(1780)\gamma) > \mathcal{B}(B \rightarrow K_4^*(2045)\gamma) \\
&> \mathcal{B}(B \rightarrow K_5^*(2380)\gamma),
\end{aligned} \tag{54}$$

and

$$\begin{aligned}
& \mathcal{B}(B \rightarrow K_1(1650)\gamma) > \mathcal{B}(B \rightarrow K_2(1820)\gamma) \gtrsim \mathcal{B}(B \rightarrow K_2(1770)\gamma) \\
& > \mathcal{B}(B \rightarrow K_2(2250)\gamma) > \mathcal{B}(B \rightarrow K_3(2320)\gamma) > \mathcal{B}(B \rightarrow K_4(2500)\gamma) \\
& > \mathcal{B}(B \rightarrow K_5(2600?)\gamma).
\end{aligned} \tag{55}$$

It is interesting to note that we obtain $1.5\mathcal{B}(B^- \rightarrow K^*(1680)\gamma) \sim \mathcal{B}(B^- \rightarrow K^*(1410)\gamma) = (27.2 \pm 8.3) \cdot 10^{-6}$, whereas Ali, Ohl, and Mannel [32] found $7\mathcal{B}(B^- \rightarrow K^*(1680)\gamma) \sim \mathcal{B}(B^- \rightarrow K^*(1410)\gamma) \simeq (35 \pm 7) \cdot 10^{-6}$.

The total branching fractions of radiative B meson decays involving resonant strange mesons¹ listed in Table V, together with $\mathcal{B}(B \rightarrow K^*(892)\gamma, K_1(1270)\gamma, K_1(1400)\gamma)$ [12–14, 35], are

$$\sum_{J=1}^5 \mathcal{B}(\bar{B}^0 \rightarrow \bar{K}_J^{(*)0} \gamma; E_\gamma^B \gtrsim 2.0 \text{ GeV}) = (237_{-34}^{+40}) \times 10^{-6}, \tag{56}$$

$$\sum_{J=1}^5 \mathcal{B}(B^- \rightarrow K_J^{(*)-} \gamma; E_\gamma^B \gtrsim 2.0 \text{ GeV}) = (252_{-36}^{+44}) \times 10^{-6}, \tag{57}$$

where E_γ^B is the photon energy in the B rest frame. Our result may hint at that the total branching fraction for the radiative B decays with (nonresonant) two-body or three-body hadronic final states is about 100×10^{-6} (see also Ref. [30]), compared to the inclusive $B \rightarrow X_s \gamma$ data [23–25]

$$\mathcal{B}(B \rightarrow X_s \gamma; E_\gamma^B > 1.7 \text{ GeV}) = (352 \pm 25) \times 10^{-6}. \tag{58}$$

The q^2 -dependence of form factors can be estimated by using the QCD counting rules [11, 40]. We consider the Breit frame, where the B meson and final state $K_J^{(*)}$ move in the opposite directions but with the same magnitude of the momentum. In the large recoil region, where $q^2 \sim 0$, since the two quarks in mesons have to interact strongly with each other to turn around the spectator quark, the transition amplitude is dominated by the one-gluon exchange between the quark-antiquark pair and is therefore proportional to $1/E^2$. Thus we get $\langle K_J^*(p_{K_J^*}, \pm 1) | V^\mu | B(p_B) \rangle \propto \epsilon^{\mu\nu\rho\sigma} p_{B\nu} p_{K_J^*\rho} \varepsilon_{(J)}^*(\pm)_\sigma \times 1/E^2$ and $\langle K_J^*(p_{K_J^*}, 0) | A^\mu | B(p_B) \rangle \propto p_{K_J^*}^\mu \times 1/E^2$. Consequently, we have $\zeta_{\perp, \parallel}(q^2) \sim 1/E^2$

¹ We do not include decays involving 1^3G_3 and 1^3H_4 states.

TABLE V: The branching fractions of the $B \rightarrow K_J^{(*)}\gamma$ decays in units of 10^{-6} , where the errors are mainly due to the uncertainties of form factors. The corresponding photon energies in the B rest frame are given in the last column.

$K_J^{(*)}$	J^{PC}	$n^{2S+1}L_J$	$\mathcal{B}(B^- \rightarrow K_J^{(*)-}\gamma)$	$\mathcal{B}(\bar{B}^0 \rightarrow \bar{K}_J^{(*)0}\gamma)$	E_γ^B [GeV]
$K^*(1410)$	1^{--}	$2^3S_1?$	27.2 ± 8.3	25.0 ± 7.7	2.45
$K^*(1680)$	1^{--}	1^3D_1	17.8 ± 8.2	16.4 ± 7.6	2.36
$K_2^*(1430)$	2^{++}	1^3P_2	13.5 ± 4.1	12.4 ± 3.8	2.45
$K_2^*(1980)$	$2^{+?}$	1^3F_2 or $2^3P_2?$	5.5 ± 3.1	5.1 ± 2.9	2.27
$K_3^*(1780)$	3^{--}	1^3D_3	4.3 ± 2.1	3.9 ± 1.9	2.34
$K_4^*(2045)$	4^{++}	1^3F_4	1.4 ± 0.8	1.3 ± 0.8	2.24
$K_5^*(2380)$	$5^{-?}$	1^3G_5	0.4 ± 0.3	0.3 ± 0.3	2.10
$K_1(1650)$	$1^{+?}$	2^1P_1 or $2^3P_1?$	18.3 ± 8.4	16.9 ± 7.8	2.38
$K_2(1770)$	2^{-+}	1^1D_2	8.0 ± 3.9	7.4 ± 3.6	2.34
$K_2(1820)$	2^{--}	$1^3D_2?$	8.5 ± 3.9	7.9 ± 3.6	2.33
$K_2(2250)$	$2^{-?}$	2^1D_2	3.0 ± 2.2	2.8 ± 2.0	2.16
$K_3(2320)$	$3^{+?}$	1^1F_3 or $1^3F_3?$	1.4 ± 1.1	1.3 ± 1.0	2.13
$K_4(2500)$	$4^{-?}$	1^1G_4 or $1^3G_4?$	0.5 ± 0.4	0.5 ± 0.3	2.05
$K_5(2600?)$	$5^{+?}$	1^1H_5 or $1^3H_5?$	0.2 ± 0.2	0.2 ± 0.2	2.00
Total ^a			135.9 ± 18.9	125.2 ± 17.4	

^aWe have assumed that $\mathcal{B}(B \rightarrow 2^1P_1\gamma) \simeq \mathcal{B}(B \rightarrow 2^3P_1\gamma)$ if 2^1P_1 and 2^3P_1 states do not mix. Analogously, we also assume that $\mathcal{B}(B \rightarrow 1^3F_2\gamma) \approx \mathcal{B}(B \rightarrow 2^3P_2\gamma)$, $\mathcal{B}(B \rightarrow 1^1F_3\gamma) \approx \mathcal{B}(B \rightarrow 1^3F_3\gamma)$, $\mathcal{B}(B \rightarrow 1^1G_4\gamma) \approx \mathcal{B}(B \rightarrow 1^3G_4\gamma)$ and $\mathcal{B}(B \rightarrow 1^1H_4\gamma) \approx \mathcal{B}(B \rightarrow 1^3H_4\gamma)$. The summation of the branching fractions should be independent of the mixture due to the unitarity. Here we do not include decays involving 1^3G_3 and 1^3H_4 states.

in the large recoil region. In other words, we can obtain that approximate forms: $\zeta_{\perp,\parallel}^{K_J^{(*)}}(q^2) = \zeta_{\perp,\parallel}^{K_J^{(*)}}(0) \cdot (1 - q^2/m_B^2)^{-2}$. This result is consistent with that obtained by Charles, Yaouanc, Oliver, Pène and Raynal [10]. They used the light-cone sum rule method to show that the $B \rightarrow V$ LEET parameters satisfy $1/E^2$ scaling law, where $V \equiv$

vector meson. Essentially, their result is also suitable for the present case.

B. $B \rightarrow K_J^{(*)} \ell^+ \ell^-$ Decays

The decay amplitude for $\bar{B} \rightarrow \bar{K}_J^{*} \ell^+ \ell^-$ is given by² [11]

$$\mathcal{M} = -i \frac{G_F \alpha_{EM}}{2\sqrt{2}\pi} V_{ts}^* V_{tb} m_B \left[\mathcal{T}_\mu^{K_J^*} \bar{s} \gamma^\mu b + \mathcal{U}_\mu^{K_J^*} \bar{s} \gamma^\mu \gamma_5 b \right], \quad (59)$$

where

$$\mathcal{T}_\mu^{K_J^*} = \mathcal{A}^{(K_J^*)} \epsilon_{\mu\nu\rho\sigma} \tilde{\epsilon}_{(J)}^*{}^\nu p_B^\rho p_{K_J^*}^\sigma - im_B^2 \mathcal{B}^{(K_J^*)} \tilde{\epsilon}_{(J)\mu}^* + i\mathcal{C}^{(K_J^*)} (\tilde{\epsilon}_{(J)}^* \cdot p_B) p_\mu + i\mathcal{D}^{(K_J^*)} (\tilde{\epsilon}_{(J)}^* \cdot p_B) q_\mu, \quad (60)$$

$$\mathcal{U}_\mu^{K_J^*} = \mathcal{E}^{(K_J^*)} \epsilon_{\mu\nu\rho\sigma} \tilde{\epsilon}_{(J)}^*{}^\nu p_B^\rho p_{K_J^*}^\sigma - im_B^2 \mathcal{F}^{(K_J^*)} \tilde{\epsilon}_{(J)\mu}^* + i\mathcal{G}^{(K_J^*)} (\tilde{\epsilon}_{(J)}^* \cdot p_B) p_\mu + i\mathcal{H}^{(K_J^*)} (\tilde{\epsilon}_{(J)}^* \cdot p_B) q_\mu, \quad (61)$$

with $q_\mu \equiv p_B - p_{K_J^*}$. The $\mathcal{D}^{(K_J^*)}$ -term vanishes when equations of motion of leptons are taken into account. The building blocks, $\mathcal{A}^{(K_J^*)}, \dots$, and $\mathcal{H}^{(K_J^*)}$ are given by

$$\mathcal{A}^{(K_J^*)} = \frac{2}{1 + \hat{m}_{K_J^{(*)}}} c_9^{\text{eff}}(\hat{s}) V^{K_J^*}(s) + \frac{4\hat{m}_b}{\hat{s}} c_7^{\text{eff}} T_1^{K_J^*}(s), \quad (62)$$

$$\mathcal{B}^{(K_J^*)} = (1 + \hat{m}_{K_J^{(*)}}) \left[c_9^{\text{eff}}(\hat{s}) A_1^{K_J^*}(s) + 2 \frac{\hat{m}_b}{\hat{s}} (1 - \hat{m}_{K_J^{(*)}}) c_7^{\text{eff}} T_2^{K_J^*}(s) \right], \quad (63)$$

$$\mathcal{C}^{(K_J^*)} = \frac{1}{1 - \hat{m}_{K_J^{(*)}}} \left[(1 - \hat{m}_{K_J^{(*)}}) c_9^{\text{eff}}(\hat{s}) A_2^{K_J^*}(s) + 2\hat{m}_b c_7^{\text{eff}} \left(T_3^{K_J^*}(s) + \frac{1 - \hat{m}_{K_J^{(*)}}}{\hat{s}} T_2^{K_J^*}(s) \right) \right], \quad (64)$$

$$\begin{aligned} \mathcal{D}^{(K_J^*)} = \frac{1}{\hat{s}} & \left[c_9^{\text{eff}}(\hat{s}) \{ (1 + \hat{m}_{K_J^{(*)}}) A_1^{K_J^*}(s) - (1 - \hat{m}_{K_J^{(*)}}) A_2^{K_J^*}(s) \} \right. \\ & \left. - 2\hat{m}_{K_J^{(*)}} A_0^{K_J^*}(s) - 2\hat{m}_b c_7^{\text{eff}} T_3^{K_J^*}(s) \right], \end{aligned} \quad (65)$$

$$\mathcal{E}^{(K_J^*)} = \frac{2}{1 + \hat{m}_{K_J^{(*)}}} c_{10} V^{K_J^*}(s), \quad \mathcal{F}^{(K_J^*)} = (1 + \hat{m}_{K_J^{(*)}}) c_{10} A_1^{K_J^*}(s), \quad (66)$$

$$\mathcal{G}^{(K_J^*)} = \frac{1}{1 + \hat{m}_{K_J^{(*)}}} c_{10} A_2^{K_J^*}(s), \quad (67)$$

$$\mathcal{H}^{(K_J^*)} = \frac{1}{\hat{s}} c_{10} \left[(1 + \hat{m}_{K_J^{(*)}}) A_1^{K_J^*}(s) - (1 - \hat{m}_{K_J^{(*)}}) A_2^{K_J^*}(s) - 2\hat{m}_{K_J^{(*)}} A_0^{K_J^*}(s) \right], \quad (68)$$

² For the amplitudes of $\bar{B} \rightarrow \bar{K}_J \ell^+ \ell^-$ decays, perform the following substitutions: $V^{K_J^*} \rightarrow A^{K_J}$, $A_i^{K_J^*} \rightarrow V_i^{K_J}$ and $T_i^{K_J^*} \rightarrow T_i^{K_J}$. The result for the decay amplitude for $\bar{B} \rightarrow \bar{K}^*(892) \ell^+ \ell^-$ can be found in Ref. [8].

where $\hat{s} = s/m_B^2$, $\hat{m}_{K_j^*} = m_{K_j^*}/m_B$, $\hat{m}_b = m_b/m_B$ and $c_9^{\text{eff}}(\hat{s}) = c_9 + Y_{\text{pert}}(\hat{s}) + Y_{\text{LD}}(\hat{s})$ with the perturbative $Y_{\text{pert}}(\hat{s})$ and long-distance $Y_{\text{LD}}(\hat{s})$ corrections [41–43]. $Y(\hat{s})_{\text{LD}}$ involves $B \rightarrow K_j^* V(\bar{c}c)$ resonances, where $V(\bar{c}c)$ are the vector charmonium states [42, 43]

$$Y_{\text{LD}}(\hat{s}) = -\frac{3\pi}{\alpha_{EM}^2} c_0 \sum_{V=\psi(1s), \dots} \kappa_V \frac{\hat{m}_V \mathcal{B}(V \rightarrow \ell^+ \ell^-) \hat{\Gamma}_{\text{tot}}^V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_{\text{tot}}^V}, \quad (69)$$

with $\hat{\Gamma}_{\text{tot}}^V \equiv \Gamma_{\text{tot}}^V/m_B$. The relevant parameters can be found in Ref. [37].

The longitudinal, transverse, and total differential decay widths are respectively given by

$$\frac{d\Gamma_L}{d\hat{s}} \equiv \frac{d\Gamma}{d\hat{s}} \Big|_{\substack{\alpha_L = \alpha_L^{(j)} \\ \beta_T = 0}}, \quad \frac{d\Gamma_T}{d\hat{s}} \equiv \frac{d\Gamma}{d\hat{s}} \Big|_{\substack{\alpha_L = 0 \\ \beta_T = \beta_T^{(j)}}}, \quad \frac{d\Gamma_{\text{total}}}{d\hat{s}} \equiv \frac{d\Gamma}{d\hat{s}} \Big|_{\substack{\alpha_L = \alpha_L^{(j)} \\ \beta_T = \beta_T^{(j)}}}, \quad (70)$$

with

$$\begin{aligned} \frac{d\Gamma}{d\hat{s}} = & \frac{G_F^2 \alpha_{EM}^2 m_B^5}{2^{10} \pi^5} |V_{ts}^* V_{tb}|^2 \\ & \times \left\{ \frac{1}{6} |\mathcal{A}^{(K_j^*)}|^2 \hat{u}(s) \hat{s} \beta_T^2 \left\{ 3 \left[1 - 2(\hat{m}_{K_j^*}^2 + \hat{s}) + (\hat{m}_{K_j^*}^2 - \hat{s})^2 \right] - \hat{u}(s)^2 \right\} \right. \\ & + \beta_T^2 |\mathcal{E}^{(K_j^*)}|^2 \hat{s} \frac{\hat{u}(s)^3}{3} \\ & + \frac{1}{12 \hat{m}_{K_j^*}^2 \lambda} |\mathcal{B}^{(K_j^*)}|^2 \hat{u}(s) \left\{ 3 \left[1 - 2(\hat{m}_{K_j^*}^2 + \hat{s}) + (\hat{m}_{K_j^*}^2 - \hat{s})^2 \right] - \hat{u}(s)^2 \right\} \\ & \quad \times \left[(-1 + \hat{m}_{K_j^*}^2 + \hat{s})^2 \alpha_L^2 + 8 \hat{m}_{K_j^*}^2 \hat{s} \beta_T^2 \right] \\ & + \frac{1}{12 \hat{m}_{K_j^*}^2 \lambda} |\mathcal{F}^{(K_j^*)}|^2 \hat{u}(s) \left\{ 3 \alpha_L^2 \lambda^2 \right. \\ & \quad \left. + \hat{u}(s)^2 \left[16 \hat{m}_{K_j^*}^2 \hat{s} \beta_T^2 - (1 - 2(\hat{m}_{K_j^*}^2 + \hat{s}) + \hat{m}_{K_j^*}^4 + \hat{s}^2 - 10 \hat{m}_{K_j^*}^2 \hat{s}) \alpha_L^2 \right] \right\} \\ & + \alpha_L^2 \hat{u}(s) \frac{\lambda}{4 \hat{m}_{K_j^*}^2} \left[|\mathcal{C}^{(K_j^*)}|^2 \left(\lambda - \frac{\hat{u}(s)^2}{3} \right) + |\mathcal{G}^{(K_j^*)}|^2 \left(\lambda - \frac{\hat{u}(s)^2}{3} + 4 \hat{m}_\ell^2 (2 + 2 \hat{m}_{K_j^*}^2 - \hat{s}) \right) \right] \\ & - \alpha_L^2 \hat{u}(s) \frac{1}{2 \hat{m}_{K_j^*}^2} \left[\text{Re}(\mathcal{B}^{(K_j^*)} \mathcal{C}^{(K_j^*)*}) \left(\lambda - \frac{\hat{u}(s)^2}{3} \right) (1 - \hat{m}_{K_j^*}^2 - \hat{s}) \right. \\ & \quad \left. + \text{Re}(\mathcal{F} \mathcal{G}^*) \left\{ \left(\lambda - \frac{\hat{u}(s)^2}{3} \right) (1 - \hat{m}_{K_j^*}^2 - \hat{s}) + 4 \hat{m}_\ell^2 \lambda \right\} \right] \\ & - 2 \alpha_L^2 \hat{u}(s) \frac{\hat{m}_\ell^2}{\hat{m}_{K_j^*}^2} \lambda \left[\text{Re}(\mathcal{F}^{(K_j^*)} \mathcal{H}^{(K_j^*)*}) - \text{Re}(\mathcal{G}^{(K_j^*)} \mathcal{H}^{(K_j^*)*}) (1 - \hat{m}_{K_j^*}^2) \right] \\ & \left. + \alpha_L^2 \hat{u}(s) \frac{\hat{m}_\ell^2}{\hat{m}_{K_j^*}^2} \hat{s} \lambda |\mathcal{H}^{(K_j^*)}|^2 \right\}. \quad (71) \end{aligned}$$

Here $\hat{u} \equiv u/m_B^2$ and $\hat{u}(s) \equiv u(s)/m_B^2$, where $u = -u(s) \cos \theta$,

$$u(s) \equiv \sqrt{\lambda \left(1 - \frac{4\hat{m}_\ell^2}{\hat{s}}\right)}, \quad (72)$$

$$\lambda \equiv 1 + \hat{m}_{K_J^*}^4 + \hat{s}^2 - 2\hat{m}_{K_J^*}^2 - 2\hat{s} - 2\hat{m}_{K_J^*}^2 \hat{s}, \quad (73)$$

and θ is the angle between the moving directions of ℓ^+ and B meson in the center of mass frame of the $\ell^+\ell^-$ pair. We show the decay distributions $d\mathcal{B}(\overline{B}^0 \rightarrow \overline{K}_J^{(*)0} \mu^+ \mu^-)/ds$ in Fig. 1 and summarize the corresponding branching fractions in Table VI. Because the decays involving heavier K -resonances have the smaller phase spaces and LEET form factors and because the Clebsch-Gordan coefficients, $\alpha_L^{(J)}$ and $\beta_T^{(J)}$, are smaller for a larger spin J , we obtain the following salient features:

$$\begin{aligned} & \mathcal{B}(B \rightarrow K^*(1410)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_2^*(1430)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K^*(1680)\mu^+\mu^-) \\ & > \mathcal{B}(B \rightarrow K_2^*(1980)\mu^+\mu^-) \approx \mathcal{B}(B \rightarrow K_3^*(1780)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_4^*(2045)\mu^+\mu^-) \\ & > \mathcal{B}(B \rightarrow K_5^*(2380)\mu^+\mu^-), \end{aligned} \quad (74)$$

and

$$\begin{aligned} & \mathcal{B}(B \rightarrow K_1(1650)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_2(1770)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_2(1820)\mu^+\mu^-) \\ & > \mathcal{B}(B \rightarrow K_2(2250)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_3(2320)\mu^+\mu^-) > \mathcal{B}(B \rightarrow K_4(2500)\mu^+\mu^-) \\ & > \mathcal{B}(B \rightarrow K_5(2600?)\mu^+\mu^-). \end{aligned} \quad (75)$$

In Fig. 2, we plot the longitudinal fraction distributions for the $\overline{B} \rightarrow \overline{K}_J^{(*)} \mu^+ \mu^-$ decays, where

$$\frac{dF_L}{ds} \equiv \frac{d\Gamma_L}{ds} \bigg/ \frac{d\Gamma_{\text{total}}}{ds}. \quad (76)$$

Our result indicates that the longitudinal fraction distribution dF_L/ds about 0.8 at $s = 2 \text{ GeV}^2$, which also apply to the inclusive process. It is interesting to note that, for the new-physics models with the flipped sign solution for c_7^{eff} , dF_L/ds can be reduced to be ~ 0.6 at $s = 2 \text{ GeV}^2$.

FIG. 1: Decay distributions of $\bar{B}^0 \rightarrow \bar{K}_J^{(*)0} \mu^+ \mu^-$ decays. The processes involving the confirmed $K_J^{(*)}$ are plotted. Solid [red], dashed [orange], dotted [green], dot-dashed [blue], and double-dot-dashed [black] curves from up to down correspond to $K_J^{(*)} = K^*(1680)$, $K_2^*(1430)$, $K_2(1770)$, $K_3^*(1780)$, and $K_4^*(2045)$, respectively. The thick and thin curves stand for the decay widths with and without charmonium resonances, respectively (see Eq. (69)).

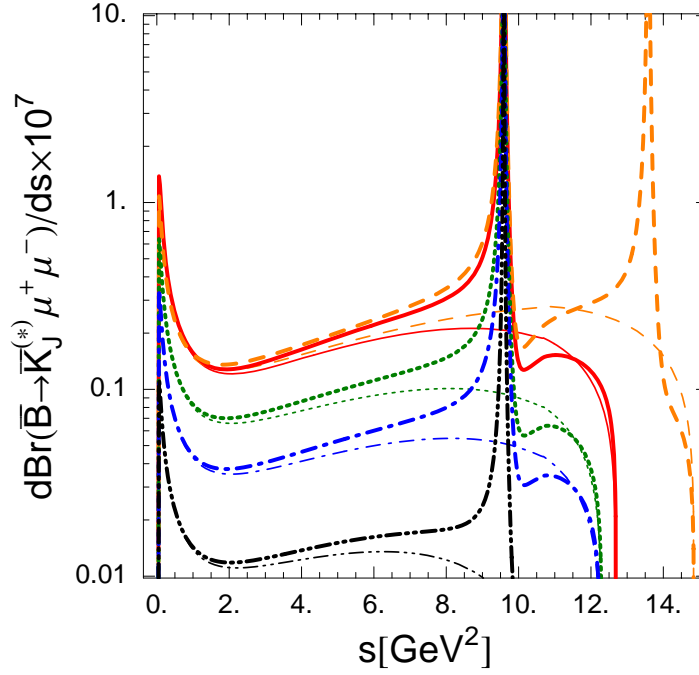


FIG. 2: Longitudinal fraction distributions dF_L/ds of $\bar{B} \rightarrow \bar{K}_J^{(*)} \mu^+ \mu^-$ decays as functions of s . Solid [red], dashed [orange], dotted [green], dot-dashed [blue] and double-dot-dashed [black] curves stand for $K_J^{(*)} = K^*(1680)$, $K_2^*(1430)$, $K_2(1770)$, $K_3^*(1780)$ and $K_4^*(2045)$, respectively.

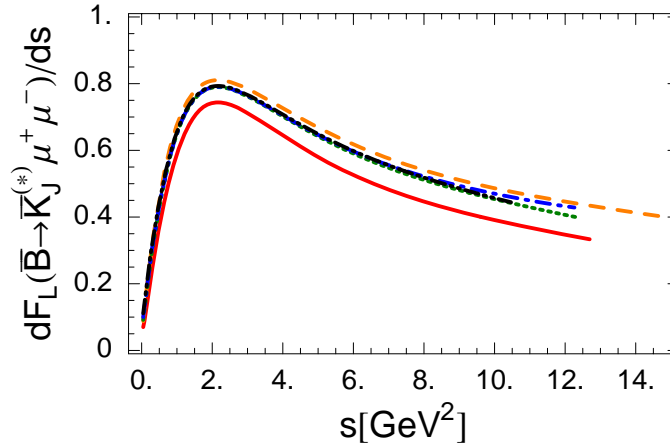


TABLE VI: Same as Table V except for nonresonant branching fractions of $\overline{B} \rightarrow \overline{K}_J^{(*)} \mu^+ \mu^-$ decays in units of 10^{-7} .

	J^{PC}	$n \ 2S+1 L_J$	$\mathcal{B}(\overline{B}^0 \rightarrow \overline{K}_J^{(*)0} \mu^+ \mu^-)$	$\mathcal{B}(B^- \rightarrow K_J^{(*)-} \mu^+ \mu^-)$
$K^*(1410)$	1^{--}	$2 \ ^3S_1$	$5.4^{+1.6}_{-1.4}$	$5.8^{+1.7}_{-1.5}$
$K^*(1680)$	1^{--}	$1 \ ^3D_1$	$2.3^{+0.8}_{-0.7}$	$2.4^{+0.9}_{-0.8}$
$K_2^*(1430)$	2^{++}	$1 \ ^3P_2$	$3.1^{+0.9}_{-0.8}$	$3.3^{+1.0}_{-0.9}$
$K_2^*(1980)$	$2^{+?}$	$1 \ ^3F_2$ or $2 \ ^3P_2$	$0.6^{+0.3}_{-0.2}$	$0.6^{+0.3}_{-0.2}$
$K_3^*(1780)$	3^{--}	$1 \ ^3D_3$	$0.6^{+0.2}_{-0.2}$	$0.6^{+0.2}_{-0.2}$
$K_4^*(2045)$	4^{++}	$1 \ ^3F_4$	$0.1^{+0.1}_{-0.1}$	$0.2^{+0.1}_{-0.1}$
$K_5^*(2380)$	$5^{-?}$	$1 \ ^3G_5$	$0.03^{+0.02}_{-0.01}$	$0.03^{+0.02}_{-0.01}$
$K_1(1650)$	$1^{+?}$	$2 \ ^1P_1$ or $2 \ ^3P_1$	$2.6^{+0.9}_{-0.8}$	$2.7^{+1.0}_{-0.8}$
$K_2(1770)$	2^{-+}	$1 \ ^1D_2$	$1.1^{+0.4}_{-0.3}$	$1.2^{+0.4}_{-0.4}$
$K_2(1820)$	2^{--}	$1 \ ^3D_2?$	$0.9^{+0.4}_{-0.3}$	$1.0^{+0.4}_{-0.3}$
$K_2(2250)$	$2^{-?}$	$2 \ ^1D_2$	$0.2^{+0.1}_{-0.1}$	$0.2^{+0.1}_{-0.1}$
$K_3(2320)$	$3^{+?}$	$1 \ ^1F_3$ or $1 \ ^3F_3$	$0.1^{+0.1}_{-0.1}$	$0.1^{+0.1}_{-0.1}$
$K_4(2500)$	$4^{-?}$	$1 \ ^1G_4$ or $1 \ ^3G_4$	$0.03^{+0.02}_{-0.02}$	$0.03^{+0.02}_{-0.02}$
$K_5(2600?)$	$5^{+?}$	$1 \ ^1H_5$ or $1 \ ^3H_5?$	$0.01^{+0.01}_{-0.01}$	$0.01^{+0.01}_{-0.01}$
Total ^a			$19.9^{+6.2}_{-4.6}$	$21.4^{+7.1}_{-5.2}$

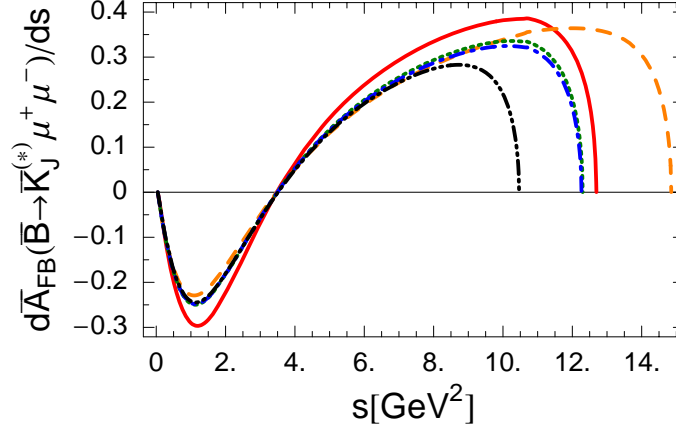
^aSame as Table V.

The forward-backward asymmetry of $\overline{B} \rightarrow \overline{K}_J^{*} \ell^+ \ell^-$ is given by

$$\begin{aligned}
\frac{dA_{FB}}{d\hat{s}} &= - \left(\beta_T^{(J)} \right)^2 \frac{G_F^2 \alpha_{EM}^2 m_B^5}{2^{10} \pi^5} |V_{ts}^* V_{tb}|^2 \hat{s} \hat{u}(s)^2 \left[\text{Re} \left(\mathcal{B}^{(K_J^*)} \mathcal{E}^{(K_J^*)*} \right) + \text{Re} \left(\mathcal{A}^{(K_J^*)} \mathcal{F}^{(K_J^*)*} \right) \right] \\
&= - \left(\beta_T^{(J)} \right)^2 \frac{G_F^2 \alpha_{EM}^2 m_B^5}{2^{10} \pi^5} |V_{ts}^* V_{tb}|^2 \hat{s} \hat{u}(s)^2 \\
&\quad \times \left[\text{Re} \left[c_{10} c_9^{\text{eff}}(\hat{s}) \right] V^{K_J^*} A_1^{K_J^*} \right. \\
&\quad \left. + \frac{\hat{m}_b}{\hat{s}} \text{Re}(c_{10} c_7^{\text{eff}}) \left\{ (1 - \hat{m}_{K_J^*}) V^{K_J^*} T_2^{K_J^*} + (1 + \hat{m}_{K_J^*}) A_1^{K_J^*} T_1^{K_J^*} \right\} \right]. \quad (77)
\end{aligned}$$

In Fig. 3 we plot the normalized forward-backward asymmetry $d\bar{A}_{FB}/ds \equiv (dA_{FB}/ds)/(d\Gamma_{\text{total}}/ds)$. Using the form factors in Eqs. (41)-(47), we can easily obtain

FIG. 3: Normalized forward-backward asymmetries for $\bar{B} \rightarrow \bar{K}_J^{(*)} \mu^+ \mu^-$ decay. Legends are the same as Fig. 2.



the forward-backward asymmetry zero, s_0 , satisfying

$$\text{Re} [c_9^{\text{eff}}(\hat{s}_0) c_{10}] = -2 \frac{\hat{m}_b}{\hat{s}_0} \text{Re}(c_7^{\text{eff}} c_{10}) \frac{1 - \hat{s}_0}{1 + \hat{m}_{K_J^{(*)}}^2 - \hat{s}_0}. \quad (78)$$

We note that s_0 is independent of the form factors but depends only on $m_{K_J^{(*)}}$. Under the variation of $\hat{m}_{K_J^{(*)}}^2$, we get

$$\delta \hat{s}_0 \simeq \frac{(\hat{s}_0 - 1) \hat{s}_0}{(\hat{s}_0 - 1)^2 + \hat{m}_{K_J^{(*)}}^2} \delta \hat{m}_{K_J^{(*)}}^2, \quad (79)$$

or

$$\delta s_0 \simeq -s_0 \cdot \frac{\delta m_{K_J^{(*)}}^2}{m_B^2}. \quad (80)$$

Since $\delta m_{K_J^{(*)}}^2 \ll s_0$ and $m_{K_J^{(*)}}^2 \ll m_B^2$, we thus expect the following relation in the SM:

$$\begin{aligned} s_0^{K^{*(980)}} &\approx 3.5 \text{ GeV}^2 \gtrsim s_0^{K^{*(1410)}} \gtrsim s_0^{K_2^{*(1430)}} \gtrsim s_0^{K^{*(1680)}} \gtrsim s_0^{K_2(1770)} \gtrsim s_0^{K_3^{*(1780)}} \gtrsim s_0^{K_2(1820)} \\ &\gtrsim s_0^{K_2^{*(1980)}} \gtrsim s_0^{K_4^{*(2045)}} \gtrsim s_0^{K_2(2250)} \gtrsim s_0^{K_3(2320)} \gtrsim s_0^{K_5^{*(2380)}} \gtrsim s_0^{K_4(2500)} \gtrsim s_0^{K_5(2600?)}. \end{aligned} \quad (81)$$

C. $\bar{B} \rightarrow \bar{K}_J^{(*)} \nu \bar{\nu}$ Decays

The effective weak Hamiltonian relevant to the $\bar{B} \rightarrow \bar{K}_J^{(*)} \nu \bar{\nu}$ decays are given by

$$\mathcal{H}_{\text{eff}} = c_L \bar{s} \gamma^\mu (1 - \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu + c_R \bar{s} \gamma^\mu (1 + \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu + \text{H.c.}, \quad (82)$$

where c_L and c_R are coefficients for left- and right-handed weak hadronic currents, respectively. In the SM, $c_R^{SM} = 0$ and

$$c_L^{SM} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{EM}}{2\pi \sin^2 \theta_W} V_{tb} V_{ts}^* X(x_t) = 2.9 \times 10^{-9}, \quad (83)$$

where the detailed form of $X(x_t)$ has been given in Refs. [44, 45]. The missing invariant mass-squared distributions, corresponding to polarizations $h = 0, \pm 1$ of the final \overline{K}_J^* for $\overline{B} \rightarrow \overline{K}_J^* \nu \bar{\nu}$ decays are³,

$$\begin{aligned} \frac{d\Gamma_0}{dq^2} = & 3 \left(\alpha_L^{(J)} \right)^2 \frac{|\vec{p}'|}{48\pi^3} \frac{|c_L - c_R|^2}{m_{K_J^*}^2} \\ & \times \left[(m_B + m_{K_J^*})(m_B E' - m_{K_J^*}^2) A_1^{K_J^*}(q^2) - \frac{2m_B^2}{m_B + m_{K_J^*}} |\vec{p}'|^2 A_2^{K_J^*}(q^2) \right]^2, \end{aligned} \quad (84)$$

$$\begin{aligned} \frac{d\Gamma_{\pm 1}}{dq^2} = & 3 \left(\beta_T^{(J)} \right)^2 \frac{|\vec{p}'| q^2}{48\pi^3} \\ & \times \left| (c_L + c_R) \frac{2m_B |\vec{p}'|}{m_B + m_{K_J^*}} V^{K_J^*}(q^2) \mp (c_L - c_R)(m_B + m_{K_J^*}) A_1^{K_J^*}(q^2) \right|^2, \end{aligned} \quad (85)$$

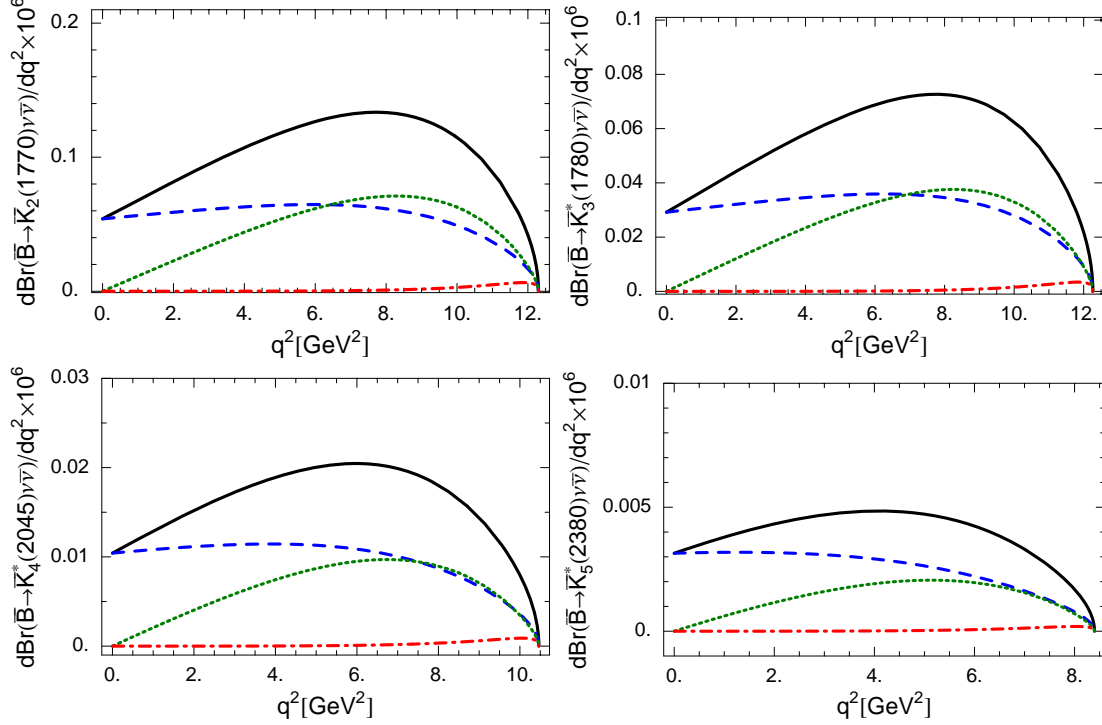
where the factor 3 counts the numbers of the neutrino generations, (E', \vec{p}') is the \overline{K}_J^* energy-momentum in the B -meson rest frame, and q^2 is the invariant mass squared of the neutrino-antineutrino pair with $0 \leq q^2 \leq (m_B - m_{K_J^*})^2$. In Fig. 4, we show the differential distributions as functions of the missing invariant mass squared in the SM. The results for branching fractions are summarized in Table VII. At $q^2 = 0$, where the neutrino and antineutrino are nearly collinear in the B rest frame, the decay is predominated by the zero helicity amplitude. Moreover, as expected from the left-handed $b_L \rightarrow s_L$ transition in the SM, $d\Gamma_+/dq^2$ is always suppressed at least by $(m_s/m_b)^2$, compared with $d\Gamma_0/dq^2$ and $d\Gamma_-/dq^2$. We obtain the relation: $d\Gamma_0/dq^2 > d\Gamma_-/dq^2 \gg d\Gamma_+/dq^2$.

IV. SUMMARY

We have formulated $B \rightarrow K_J^{(*)}$ form factors using large energy effective theory techniques. We have studied the radiative and semileptonic B decays involving the higher strange resonance $K_J^{(*)}$ in the final state. The main results are as follows.

³ For the amplitudes of $\overline{B} \rightarrow \overline{K}_J^* \nu \bar{\nu}$ decays, perform the following replacements: $V^{K_J^*} \rightarrow A^{K_J^*}$, $A_i^{K_J^*} \rightarrow V_i^{K_J^*}$.

FIG. 4: The $d\mathcal{B}(\bar{B} \rightarrow \bar{K}_J^{(*)} \nu \bar{\nu})/dq^2$ as functions of the missing invariant mass-squared q^2 . The solid [black], dashed [blue], dotted [green] and dot-dashed [red] curves correspond to the total decay rate and the polarization rates with helicities $h = 0, -1, +1$, respectively.



- The transition form factors in the large recoil region can be represented in terms of two independent LEET form factors, $\zeta_{\perp}^{K_J^{(*)}}(q^2)$ and $\zeta_{\parallel}^{K_J^{(*)}}(q^2)$. According to the QCD counting rules, these two form factors exhibit the dipole q^2 dependence in the large recoil region (and in the LEET limit). We have further estimated $\zeta_{\perp}^{K_J^{(*)}}(0)$ and $\zeta_{\parallel}^{K_J^{(*)}}(0)$ in the BSW model.
- The branching fractions for decays $\bar{B} \rightarrow \bar{K}_J^{(*)} \gamma$, $\bar{B} \rightarrow \bar{K}_J^{(*)} \ell^+ \ell^-$ and $\bar{B} \rightarrow \bar{K}_J^{(*)} \nu \bar{\nu}$ with higher K -resonances are suppressed due to the smaller phase space and $\zeta_{\perp, \parallel}^{K_J^{(*)}}$, and/or due to the smaller Clebsch-Gordan coefficients, $\beta_T^{(J)}$ and $\alpha_L^{(J)}$, in case of larger spin- J .
- We find that for $\bar{B} \rightarrow \bar{K}_J^{(*)} \ell^+ \ell^-$ decays, the longitudinal fraction distribution $dF_L/ds \simeq 0.8$ at $s = 2 \text{ GeV}^2$, and the forward-backward asymmetry zero $s_0 \simeq 3.5 \text{ GeV}^2$. The asymmetry zero is independent of the form factors in the LEET limit and highly insensitive to $m_{K_J^{(*)}}$.

TABLE VII: The branching fractions of the $B \rightarrow K_J^{(*)} \nu \bar{\nu}$ decays in units of 10^{-6} . The first and second errors correspond to the uncertainties of the form factors $\zeta_{\perp}^{K_J^{(*)}}$ and $\xi^{K_J^{(*)}}$, respectively.

	J^{PC}	$n^{2S+1}L_J$	$\mathcal{B}(\overline{B}^0 \rightarrow \overline{K}_J^{(*)0} \nu \bar{\nu})$	$\mathcal{B}(B^- \rightarrow K_J^{(*)-} \nu \bar{\nu})$
$K^*(1410)$	1^{--}	$2^3S_1?$	$4.3_{-1.1}^{+1.3}$	$4.6_{-1.2}^{+1.4}$
$K^*(1680)$	1^{--}	1^3D_1	$1.8_{-0.6}^{+0.7}$	$2.0_{-0.6}^{+0.7}$
$K_2^*(1430)$	2^{++}	1^3P_2	$2.5_{-0.6}^{+0.7}$	$2.6_{-0.7}^{+0.8}$
$K_2^*(1980)$	$2^{+?}$	1^3F_2 or $2^3P_2?$	$0.4_{-0.2}^{+0.2}$	$0.5_{-0.2}^{+0.2}$
$K_3^*(1780)$	3^{--}	1^3D_3	$0.5_{-0.2}^{+0.2}$	$0.5_{-0.2}^{+0.2}$
$K_4^*(2045)$	4^{++}	1^3F_4	$0.11_{-0.04}^{+0.05}$	$0.11_{-0.05}^{+0.06}$
$K_5^*(2380)$	$5^{-?}$	1^3G_5	$0.02_{-0.01}^{+0.01}$	$0.02_{-0.01}^{+0.01}$
$K_1(1650)$	$1^{+?}$	2^1P_1 or $2^3P_1?$	$2.1_{-0.6}^{+0.7}$	$2.2_{-0.7}^{+0.8}$
$K_2(1770)$	2^{-+}	1^1D_2	$0.9_{-0.3}^{+0.3}$	$0.9_{-0.3}^{+0.4}$
$K_2(1820)$	2^{--}	$1^3D_2?$	$0.7_{-0.2}^{+0.3}$	$0.8_{-0.3}^{+0.3}$
$K_2(2250)$	$2^{-?}$	2^1D_2	$0.2_{-0.1}^{+0.1}$	$0.2_{-0.1}^{+0.1}$
$K_3(2320)$	$3^{+?}$	1^1F_3 or 1^3F_3	$0.07_{-0.03}^{+0.05}$	$0.07_{-0.04}^{+0.05}$
$K_4(2500)$	$4^{-?}$	1^1G_4 or 1^3G_4	$0.02_{-0.01}^{+0.02}$	$0.02_{-0.01}^{+0.01}$
$K_5(2600)$	$5^{+?}$	1^1H_5 or 1^3H_5	$0.008_{-0.005}^{+0.006}$	$0.008_{-0.005}^{+0.007}$
Total ^a			$16.2_{-3.0}^{+4.1}$	$17.3_{-3.5}^{+4.7}$

^aSame as Table V.

- For the $\overline{B} \rightarrow \overline{K}_J^{(*)} \nu \bar{\nu}$ decay, the branching fraction is predominated by the zero helicity amplitude at $q^2 = 0$, where the neutrino and antineutrino are nearly collinear in the B rest frame. As expected from the left-handed $b_L \rightarrow s_L$ current in the SM, $d\Gamma_+/dq^2$ is always suppressed at least by $(m_s/m_b)^2$, compared with $d\Gamma_0/dq^2$ and $d\Gamma_-/dq^2$. We thus predict the relation: $d\Gamma_0/dq^2 > d\Gamma_-/dq^2 \gg d\Gamma_+/dq^2$.

Acknowledgments

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Appendix A: $\overline{B} \rightarrow \overline{K}_J$ form factors

$\overline{B} \rightarrow \overline{K}_J$ transition form factors in the LEET limit are given by

$$\langle \overline{K}_J | A^\mu | \overline{B} \rangle = -i2E \left(\frac{m_{K_J}}{E} \right)^{J-1} \zeta_\perp^{K_J(a)} \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho e_\sigma^*, \quad (\text{A1})$$

$$\begin{aligned} \langle \overline{K}_J | V^\mu | \overline{B} \rangle &= 2E \left(\frac{m_{K_J}}{E} \right)^{J-1} \zeta_\perp^{K_J(v)} [e^{*\mu} - (e^* \cdot v) n^\mu] \\ &\quad + 2E \left(\frac{m_{K_J}}{E} \right)^J (e^* \cdot v) \left[\zeta_\parallel^{K_J(v)} n^\mu + \zeta_{\parallel,1}^{K_J(v)} v^\mu \right], \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \langle \overline{K}_J | T_A^{\mu\nu} | \overline{B} \rangle &= -2E \left(\frac{m_{K_J}}{E} \right)^J \zeta_\parallel^{K_J(t_5)} (e^* \cdot v) \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \\ &\quad - 2E \left(\frac{m_{K_J}}{E} \right)^{J-1} \zeta_\perp^{K_J(t_5)} \epsilon^{\mu\nu\rho\sigma} n_\rho [e_\sigma^* - (e^* \cdot v) n_\sigma] \\ &\quad - 2E \left(\frac{m_{K_J}}{E} \right)^{J-1} \zeta_{\perp,1}^{K_J(t_5)} \epsilon^{\mu\nu\rho\sigma} v_\rho [e_\sigma^* - (e^* \cdot v) n_\sigma], \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \langle \overline{K}_J | T^{\mu\nu} | \overline{B} \rangle &= i2E \left(\frac{m_{K_J}}{E} \right)^{J-1} \zeta_{\perp,1}^{K_J(t)} \{ [e^{*\mu} - (e^* \cdot v) n^\mu] v^\nu - (\mu \leftrightarrow \nu) \} \\ &\quad + i2E \left(\frac{m_{K_J}}{E} \right)^{J-1} \zeta_\perp^{K_J(t)} \{ [e^{*\mu} - (e^* \cdot v) n^\mu] n^\nu - (\mu \leftrightarrow \nu) \} \\ &\quad + i2E \left(\frac{m_{K_J}}{E} \right)^J \zeta_\parallel^{K_J(t)} (e^* \cdot v) (n^\mu v^\nu - n^\nu v^\mu), \end{aligned} \quad (\text{A4})$$

where m_{K_J} is the mass of the K_J . $\langle \overline{K}_J | T_A^{\mu\nu} | \overline{B} \rangle$ is related to $\langle \overline{K}_J | T^{\mu\nu} | \overline{B} \rangle$ by the relation: $\sigma^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} = 2i\sigma^{\rho\sigma} \gamma_5$. From operator relations Eqs. (28)-(32) and

$$\bar{s}_n \gamma_5 b_v = -n_\mu \bar{s}_n \gamma^\mu \gamma_5 b_v, \quad (\text{A5})$$

we obtain

$$\zeta_\perp^{K_J(v)} = \zeta_\perp^{K_J(a)} = \zeta_\perp^{K_J(t)} = \zeta_\perp^{K_J(t_5)} \equiv \zeta_\perp^{K_J}, \quad (\text{A6})$$

$$\zeta_\parallel^{K_J(a)} = \zeta_\parallel^{K_J(t)} = \zeta_\parallel^{K_J(t_5)} \equiv \zeta_\parallel^{K_J}, \quad (\text{A7})$$

$$\zeta_{\parallel,1}^{K_J(a)} = \zeta_{\perp,1}^{K_J(t_5)} = \zeta_{\perp,1}^{K_J(t)} = 0, \quad (\text{A8})$$

and thus find that there are only two independent form factors, $\zeta_\perp^{K_J}(q^2)$ and $\zeta_\parallel^{K_J}(q^2)$.

$\overline{B} \rightarrow \overline{K}_J$ form factors are given by

$$\langle \overline{K}_J(p_{K_J}, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | \overline{B}(p_B) \rangle = -i \frac{2}{m_B + m_{K_J}} \tilde{A}^{K_J}(q^2) \epsilon^{\mu\nu\rho\sigma} p_{B\nu} p_{K_J\rho} e(\lambda)_\sigma^*, \quad (\text{A9})$$

$$\begin{aligned} \langle \overline{K}_J(p_{K_J}, \lambda) | \bar{s} \gamma^\mu b | \overline{B}(p_B) \rangle &= 2m_{K_J} \tilde{V}_0^{K_J}(q^2) \frac{e(\lambda)^* \cdot p_B}{q^2} q^\mu \\ &\quad + (m_B + m_{K_J}) \tilde{V}_1^{K_J}(q^2) \left[e(\lambda)^{* \mu} - \frac{e(\lambda)^* \cdot p_B}{q^2} q^\mu \right] \\ &\quad - \tilde{V}_2^{K_J}(q^2) \frac{e(\lambda)^* \cdot p_B}{m_B + m_{K_J}} \left[p_B^\mu + p_{K_J}^\mu - \frac{m_B^2 - m_{K_J}^2}{q^2} q^\mu \right], \end{aligned} \quad (\text{A10})$$

$$\langle \overline{K}_J(p_{K_J}, \lambda) | \bar{s} \sigma^{\mu\nu} \gamma_5 q_\nu b | \overline{B}(p_B) \rangle = 2\tilde{T}_1^{K_J}(q^2) \epsilon^{\mu\nu\rho\sigma} p_{B\nu} p_{K_J\rho} e(\lambda)_\sigma^*, \quad (\text{A11})$$

$$\begin{aligned} \langle \overline{K}_J(p_{K_J^*}, \lambda) | \bar{s} \sigma^{\mu\nu} q_\nu b | \overline{B}(p_B) \rangle &= i\tilde{T}_2^{K_J}(q^2) \left[(m_B^2 - m_{K_J}^2) e(\lambda)^{* \mu} - (e(\lambda)^* \cdot p_B) (p_B^\mu + p_{K_J}^\mu) \right] \\ &\quad + i\tilde{T}_3^{K_J}(q^2) (e(\lambda)^* \cdot p_B) \\ &\quad \times \left[q^\mu - \frac{q^2}{m_B^2 - m_{K_J}^2} (p_B^\mu + p_{K_J}^\mu) \right]. \end{aligned} \quad (\text{A12})$$

We can further obtain the following relations,

$$\tilde{V}_0^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv V_0^{K_J}(q^2) \simeq \left(1 - \frac{m_{K_J}^2}{m_B E} \right) \zeta_\parallel^{K_J}(q^2) + \frac{m_{K_J}}{m_B} \zeta_\perp^{K_J}(q^2), \quad (\text{A13})$$

$$\tilde{V}_1^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv V_1^{K_J}(q^2) \simeq \frac{2E}{m_B + m_{K_J}} \zeta_\perp^{K_J}(q^2), \quad (\text{A14})$$

$$\tilde{V}_2^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv V_2^{K_J}(q^2) \simeq \left(1 + \frac{m_{K_J}}{m_B} \right) \left[\zeta_\perp^{K_J}(q^2) - \frac{m_{K_J}}{E} \zeta_\parallel^{K_J}(q^2) \right], \quad (\text{A15})$$

$$\tilde{A}^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv A^{K_J}(q^2) \simeq \left(1 + \frac{m_{K_J}}{m_B} \right) \zeta_\perp^{K_J}(q^2), \quad (\text{A16})$$

$$\tilde{T}_1^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv T_1^{K_J}(q^2) \simeq \zeta_\perp^{K_J}(q^2), \quad (\text{A17})$$

$$\tilde{T}_2^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv T_2^{K_J}(q^2) \simeq \left(1 - \frac{q^2}{m_B^2 - m_{K_J}^2} \right) \zeta_\perp^{K_J}(q^2), \quad (\text{A18})$$

$$\tilde{T}_3^{K_J}(q^2) \left(\frac{|\vec{p}_{K_J}|}{m_{K_J}} \right)^{J-1} \equiv T_3^{K_J}(q^2) \simeq \zeta_\perp^{K_J}(q^2) - \left(1 - \frac{m_{K_J}^2}{m_B^2} \right) \frac{m_{K_J}}{E} \zeta_\parallel^{K_J}(q^2), \quad (\text{A19})$$

where use of $p_3/E \simeq 1$ has been made. Recalling that

$$\tilde{\varepsilon}_{(J)}(0)^\mu = \alpha_L^{(J)} \varepsilon(0)^\mu, \quad \tilde{\varepsilon}_{(J)}(\pm 1)^\mu = \beta_T^{(J)} \varepsilon(\pm 1)^\mu, \quad (\text{A20})$$

we can easily generalize the studies for $B \rightarrow K_J^* \gamma$, $B \rightarrow K_J^* \ell^+ \ell^-$ and $B \rightarrow K_J^* \nu \bar{\nu}$ to $B \rightarrow K_J \gamma$, $B \rightarrow K_J \ell^+ \ell^-$ and $B \rightarrow K_J \nu \bar{\nu}$ by the following replacements:

$$V^{K_J^*} \rightarrow A^{K_J}, \quad A_i^{K_J^*} \rightarrow V_i^{K_J} \quad (i = 0, 1, 2), \quad T_j^{K_J^*} \rightarrow T_j^{K_J} \quad (j = 1, 2, 3). \quad (\text{A21})$$

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